

Thermodynamic analysis of a beta-type Stirling engine with rhombic drive mechanism



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ABSTRACT

This paper presents a theoretical investigation on kinematic and thermodynamic analysis of a beta type Stirling engine with rhombic-drive mechanism. Variations in the hot and cold volumes of the engine were calculated using kinematic relations. Two different displacer cylinders were investigated: one of them had smooth inner surface and the other had axial slots grooved into the cylinder to increase the heat transfer area. The effects of the slots grooved into the displacer cylinder inner surface on the performance were calculated using nodal analysis in Fortran. The effects of working fluid mass on cyclic work were investigated using 200, 300 and 400 W/m² K convective heat transfer coefficients for smooth and grooved displacer cylinders. The variation of engine power with engine speed was obtained by using the same convective heat transfer coefficients and isothermal conditions. The convective heat transfer coefficient was predicted as 104 W/m² K using the experimental results measured from the prototype engine under atmospheric conditions. The variation in cyclic work determined by the experimental study was also compared with the theoretical results obtained for different convective heat transfer coefficients and isothermal conditions.

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1. Introduction

The Stirling engine is an external combustion machine, invented by Robert Stirling in 1816. The thermodynamic cycle of the Stirling engine consists of two isothermal and two constant volume processes [1,3–5]. Since the Stirling engine is externally heated, it can be operated with solar radiation, geothermal energy, natural gas, biomass, fossil fuels and all kinds of energy sources [2,6,7]. In Stirling engines, mostly air, helium and hydrogen are used as the working fluid [8,9]. Stirling engines have some advantages such as high thermal efficiency, low noise, low pollutant levels, and long life when compared with internal combustion engines [8–11].

Kinematic Stirling engines are classified into three different mechanical arrangements as α , β , and γ [3,5,12]. α -type engines use two pistons situated with a 90° phase angle in separate cylinders. Cylinders can be configured in the form of a V or placed as parallel to each other. In β -type Stirling engines, the cycle is conducted by a piston and a displacer operating concentrically in the same cylinder. In γ -type engines, a piston and a displacer are situated in separate cylinders. The displacer cylinder provides heating

and cooling of the working fluid. A regenerator can be placed inside or outside the displacer [3,12–14].

Different driving mechanisms such as a crankshaft, Ross-yoke, lever-drive, rhombic-drive, Scotch-yoke or swash-plate are used in Stirling engines. The cycle obtained by these driving mechanisms is different from the ideal Stirling cycle depending on the mechanical configuration. Cyclic work is the most important parameter in engine design [15]. Thermodynamic analyses are quite significant in determining the engine performance and parameters that affect the performance. Various analysis methods have been developed since the invention of Stirling engines and engine performance has been estimated during the design process.

The first analysis for determining the thermodynamic performance of Stirling engines was developed by Gustav Schmidt in 1871 and thermodynamic analyses of α , β , and γ type engines were conducted [3,6]. The nodal analysis method was developed in 1967 by Finkelstein. In this analysis, the total volume of the engine was divided into 13 nodal volumes: the expansion volume (1), heater volumes (3), regenerator volumes (5), cooler volumes (3), and cold volume (1). Instantaneous temperatures, system pressure, cyclic work, temperatures and thermal efficiency are calculated in the nodal volumes [15–17]. In 1978, Martini developed a five-zone nodal-isothermal analysis. In this model, the temperature of the working fluid in the nodal volumes was assumed to be equal to the wall temperature [18]. In 1994, Ladas and Ibrahim conducted

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Nomenclature

A_i	nodal values of heat transfer surface (m^2)	Δm_i	variation of nodal mass within a time step (kg)
c_p	specific heat at constant pressure ($\text{J kg}^{-1} \text{K}^{-1}$)	R	gas constant ($\text{J kg}^{-1} \text{K}^{-1}$)
c_v	specific heat at constant volume ($\text{J kg}^{-1} \text{K}^{-1}$)	R_{cr}	radius of crankshaft (m)
E_i	enthalpy flow in and out of the nodal volumes (J)	T_i	nodal values of working fluid temperature (K)
e_c	length of cold volume (m)	T_c	cold-end temperature of displacer cylinder (K)
e_h	length of hot volume (m)	T_H	hot-end temperature of displacer cylinder (K)
$\frac{h_p}{g}$	distance between piston top and piston rod top (m)	$T_{w,i}$	nodal values of heat transfer surface temperature (K)
h_d	displacer length (m)	t	time (s)
h_i	nodal values of convective heat transfer coefficient ($\text{Wm}^{-2} \text{K}^{-1}$)	Δt	period of time steps (s)
l_{dr}	length of displacer rod (m)	U_c	length from cylinder top to rhombic center (m)
l_{pr}	length of piston rod (m)	V_i	nodal values of volume (m^3)
l_{rh}	length of displacer and piston connecting rods (m)	βr	angle made by connecting rod with vertical (rad)
m_t	total mass of working fluid (kg)	θ	total crankshaft rotation (rad)
m_i	nodal values of working fluid mass (kg)		

a finite-time thermodynamic analysis of a Stirling engine. They analyzed the effects of regeneration and engine speed on engine power and engine efficiency by using the conservation law of mass and the energy equations [19]. In 2000, Kaushik and Kumar determined the thermodynamic performance of a Stirling engine using finite-time thermodynamics. They investigated the effects of the temperature of different heat exchangers and heat source on engine power and engine efficiency [20]. In a study conducted by Karabulut et al. (2006) the power cylinder was concentrically situated in the displacer cylinder to increase the specific power and decrease the external volume of a gamma type Stirling engines. The thermodynamic performance characteristics of the engine were predicted by a nodal analysis program in Fortran [21]. Kongtragool and Wongwises (2006) investigated the performance of a Stirling engine with the dead volumes of the cold volume, hot volume and regenerator using an isothermal model [22]. Parlak et al. (2009) conducted a thermodynamic analysis of a gamma type Stirling engine using a quasi-steady flow model. The analysis was performed for five volumes: compression, expansion, cooler, heater and regenerator. The variations of temperature, pressure, mass and work were calculated by using the conservation law of mass and the energy equations [23]. Timoumi et al. (2008) investigated the effects of regenerator sizes and materials on the performance of a General Motors GPU-3 Stirling engine [24]. Eid (2009) conducted a thermodynamic analysis of a beta type Stirling engine with a regenerative displacer by using the Schmidt analysis method [13]. Cheng and Yu (2010) carried out the thermodynamic analysis of a beta type Stirling engine with a rhombic-drive mechanism by using a numerical model. The energy equations for control volumes were derived and solved by taking into account the non-isothermal effects, the effectiveness of the regenerative channel and the thermal resistance of the heating head [12].

In this study, kinematic and thermodynamic analysis of a beta type Stirling engine with a rhombic-drive mechanism was conducted using the nodal analysis method. In the analysis, the increase of the heat transfer surface area inside the displacer cylinder on engine performance was investigated. The performance of the engine was estimated for two different displacer cylinders, of which one has a smooth inner surface and the other has axial slots grooved into the cylinder. Engine performance parameters were also calculated for different values of the convective heat transfer coefficient, working fluid mass and engine speed. The theoretical results were compared with experimental data obtained from the prototype engine.

2. Test engine

Due to the lower side thrust and more silent operation [6], the rhombic drive mechanism was preferred and the engine was designed as beta type. The cycle is conducted by a piston and a displacer operating in the same cylinder. The rhombic drive mechanism consisted of a piston yoke, a displacer yoke, two pistons and two displacer connecting rods with equal length hexagon and two symmetric spur gears of equal diameter (Fig. 1). The spur gears rotate in opposite directions. The displacer rod is connected to the displacer yoke at the lower end of the equal length hexagon; the power piston rod is connected to the power piston yoke at the upper end of the equal length hexagon. The displacer and power piston yokes have linear motion. A displacer and a power piston rods are connected to pins on spur gears. The cold volume is between the power piston and the displacer, and the hot volume is

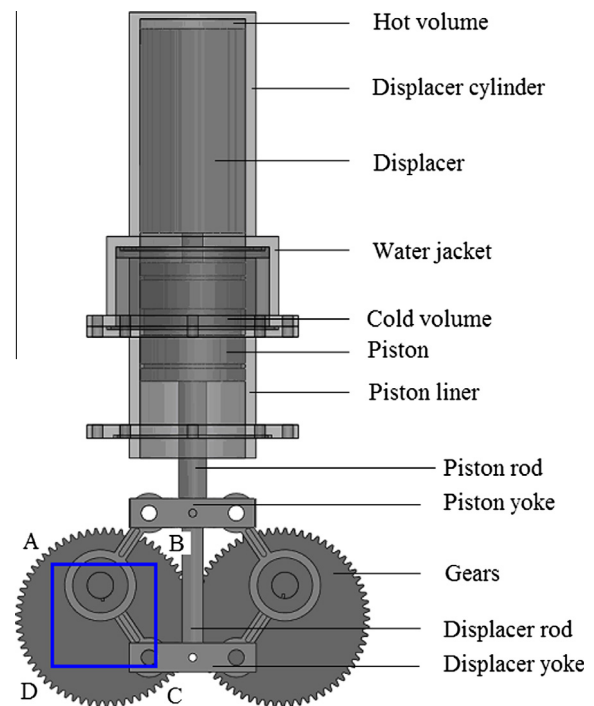


Fig. 1. Beta type Stirling engine with rhombic-drive mechanism.

Table 1

Technical specifications of the engine.

Engine type	β
Driving mechanism	Rhombic
Diameter of displacer cylinder (mm)	86
Diameter of displacer piston (mm)	84.5
Length of displacer (mm)	155
Piston diameter (mm)	86
Length of piston (mm)	90
Rhombus length (l_{rh}) (mm)	66
Radius of crankshaft (mm)	$l_{rh}/2.6666$
Displacer rod diameter (mm)	16
Hot source temperature (T_H) (K)	773
Cold source temperature (T_C) (K)	300
Working fluid	Air
Engine speed (rpm)	500

above the displacer piston. The gap between the displacer and its cylinder is used as the regenerator. The schematic illustration and technical specifications of the engine are given in Fig. 1 and Table 1.

As seen in Fig. 1, where the spur gear pin is at A, working fluid is compressed in the cold volume and the hot volume is at its minimum. At this position of the spur gear pin θ is assumed to be zero. While the spur gear pin moves from A to B, the power piston remains around the bottom dead center of its stroke. The displacer moves downwards and working fluid is heated up by being pumped from the cold volume to the hot volume. The reduction in cold volume and expansion of hot volume are equal to each other, so the total volume remains constant. Thus, this process is carried out at a constant temperature. When the spur gear pin moves from B to C, the power piston and displacer move simultaneously downward. While the cold volume remains constant at a minimum value, the working fluid expands in the hot volume by producing work. In this process, a lot of working fluid expands in the hot volume with the use of the rhombic-drive mechanism. When the spur gear pin moves from C to D, the power piston conducts a small displacement around the bottom dead center of its stroke. The displacer moves to the top dead center of its stroke. The displacer expels the working fluid from the hot volume to the cold volume, performing a cooling process at constant volume. When the spur gear pin moves from D to A, the displacer moves to the top dead center of its stroke and the hot volume is reduced. The power piston moves from down to up and the working fluid is compressed in the cold volume. Thus, the cycle is completed.

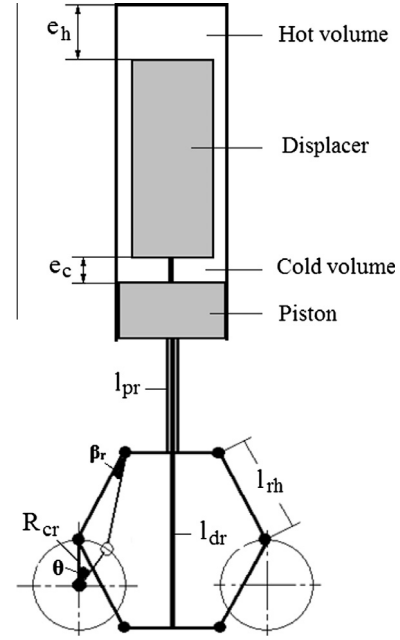
3. Kinematic relations and mathematical model

In the engine the spaces occupied by the working fluid are the hot volume, cold volume and regenerator volume. Hot volume is above the displacer top. Cold volume is between piston top and displacer bottom. Hot and cold volumes are calculated by using the kinematical relations. Fig. 2 shows the kinematic relations and schematic view of the engine with the rhombic-drive mechanism.

The regenerator volume is the gap between the outer surface of the displacer and the inner surface of the cylinder and have constant value. The total volume of the engine was divided into 50 nodal volumes: the hot volume (1), the regenerator volumes (48), and the cold volume (1). The following kinematic relations are used to calculate the values of cold and hot volumes:

$$R_{cr} = \frac{l_{rh}}{2.666} \quad (1)$$

$$\beta_r = \sin^{-1} \left(\frac{1}{2} - \frac{R_{cr}}{l_{rh}} \sin \theta \right) \quad (2)$$

**Fig. 2.** Schematic view of the engine and nomenclature.

$$e_c = (R_{cr} \cos \theta - l_{rh} \cos \beta_r + l_r) - \left(l_{pr} + R_{cr} \cos \theta + l_{rh} \cos \beta_r + \frac{H_p}{9} \right) \quad (3)$$

$$e_h = U_c - (R_{cr} \cos \theta - l_{rh} \cos \beta_r + l_{dr} + H_d) \quad (4)$$

$$V_c = e_c(A_c - A_r) \quad (5)$$

$$V_h = e_h A_h \quad (6)$$

All nodal volumes are open systems and consist of hot, cold and regenerator volumes. The working fluid friction losses between the nodal volumes are ignored and the following equation is used to calculate the pressure in the system:

$$P = \frac{m_t R}{\frac{V_1}{T_1} + \frac{V_2}{T_2} + \frac{V_3}{T_3} + \dots + \frac{V_n}{T_n}} \quad (7)$$

The temperature variation in nodal volumes is calculated using the first law of thermodynamics for open systems. The temperature variations in nodal volumes are calculated by,

$$\Delta T = \frac{[h_i A_i (T_{w,i} - T_i) \Delta t - \Delta m_i C_v T_i + E_i - p \Delta V_i]}{m_i C_v} \quad (8)$$

The following expression is used to calculate the enthalpy flow in or out of the nodal volumes:

$$E_i = -C_p \frac{T_i + T_{i+1}}{2} \sum_{j=i+1}^n \Delta m_j - C_p \frac{T_{i-1} + T_i}{2} \sum_{j=1}^{i-1} \Delta m_j \quad (9)$$

Hot and cold source temperatures are assumed to be constant. The surface temperature of the flow passage is supposed to be constant depending on time. The temperature changes linearly in the flow direction from cold source to hot source. Numerical solutions and detailed information of nodal analysis were given by Karabulut et al. [21].

4. Experimental setup

The experimental setup is shown in Fig. 3. A Prony type dynamometer with an accuracy of 0.003 Nm was used to measure en-

gine load. The dynamometer consisted of a brake disk, a load cell and a torque arm. The engine speed was measured by a digital tachometer, ENDA ETS1410, with 1 rpm accuracy. The temperatures were measured by an infrared thermometer with $\pm 2^\circ\text{C}$ accuracy and -50 to 1600°C measurement range. The experiments were carried out at a displacer cylinder hot-end temperature of 673 K . Heat was supplied by an LPG burner.

5. Results and discussion

For the thermodynamic analysis, an engine speed of 500 rpm , cold source temperature of 300 K , and hot source temperature of 773 K were determined. The inner heat transfer surface area of the displacer cylinder was enlarged by grooving 90 axial slots with 1.5 mm width and 3 mm depth. For smooth and grooved cylinders, regenerator volumes were determined as 30 cm^3 and 90 cm^3 , and heat transfer surface areas were determined as 480 cm^2 and 1200 cm^2 , respectively.

Fig. 4 shows the cold volume, hot volume and total volume changes for grooved and smooth displacer cylinder. The variations of hot and cold volumes are the same for the grooved and smooth displacer cylinders. Channels inside the cylinder affect the total volume. The regenerator volume of the grooved cylinder is 60 cm^3 higher compared to the smooth displacer cylinder. Therefore, the total volume of the engine with the grooved displacer cylinder is higher than that of the engine with the smooth displacer cylinder. As seen in Fig. 4, the interval of the crank angle from 45° to 135° corresponds to the expansion process of the working fluid at constant temperature. The minimum cold volume appears at a crank angle of 90° . Since the cold volume remains at a minimum value in this crank angle range, the working fluid expands in the hot volume. The interval of crank angle from 135° to 225° carries out the cooling process – at constant volume. In this process, hot volume decreases and cold volume increases and the total volume remains constant. The compression process occurs at constant temperature in the 225 – 315° range of the crank angle. When the compression process starts at 225° crank angle, the cold and hot volumes are equal. During this process the hot volume decreases. The cold volume increases up to the crank angle of 270° ,

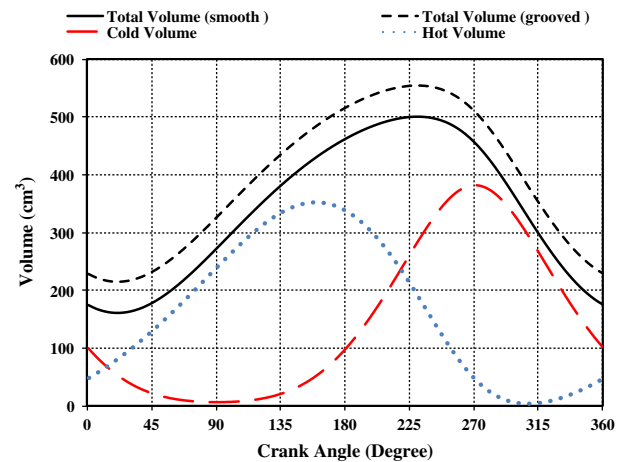


Fig. 4. Comparison of cold volume, hot volume and total volume variations obtained with grooved and smooth displacer cylinders.

and then decreases. The heating process occurs in the range of 315 – 45° crank angle at constant volume.

p – V diagrams of the engine with grooved and smooth displacer cylinders at a charge pressure of 2 bar are given in Fig. 5 for $300\text{ W/m}^2\text{ K}$ convective heat transfer coefficient and isothermal conditions. The dead volume of the engine with the grooved displacer cylinder engine is higher than the dead volume of the engine with the smooth displacer cylinder. The compression ratios of the engines with grooved and smooth displacer cylinders were calculated as 2.52 and 3.07 , respectively. For a convective heat transfer coefficient of $300\text{ W/m}^2\text{ K}$, engines with grooved and smooth displacer cylinders produce work of 21.37 and 18.4 J , respectively. The cyclic top pressure of the engine with the grooved displacer cylinder is lower compared to the engine with the smooth displacer cylinder and its working fluid leakage is less than that of the smooth displacer cylinder. Due to the increase of the heat transfer area, the work obtained by the grooved displacer cylinder is higher compared to that of the smooth displacer cylinder.

Fig. 6 shows variations of work with working fluid mass for convective heat transfer coefficients of 200 , 300 , and $400\text{ W/m}^2\text{ K}$.

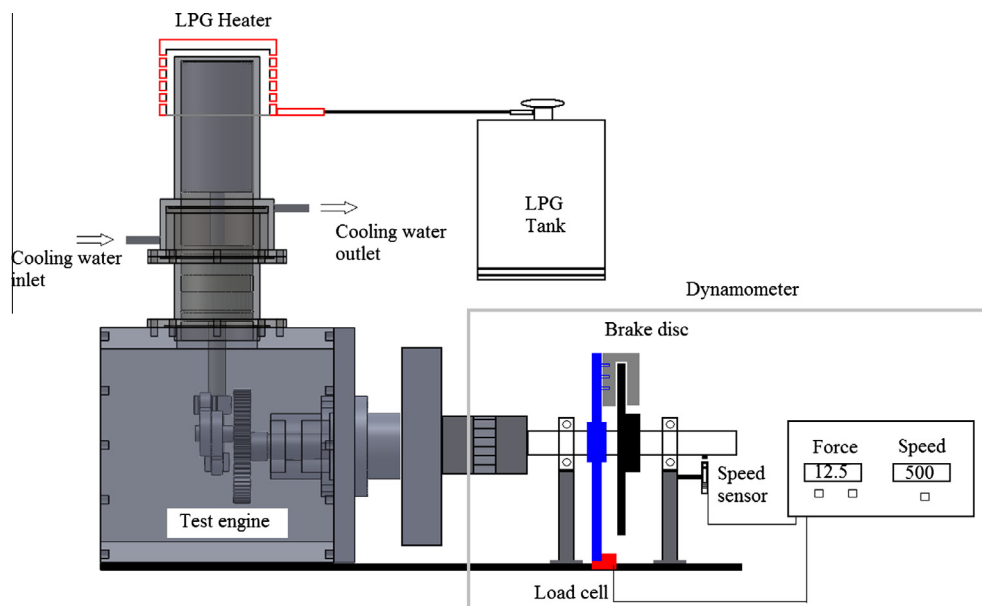


Fig. 3. Schematic view of the experimental setup.

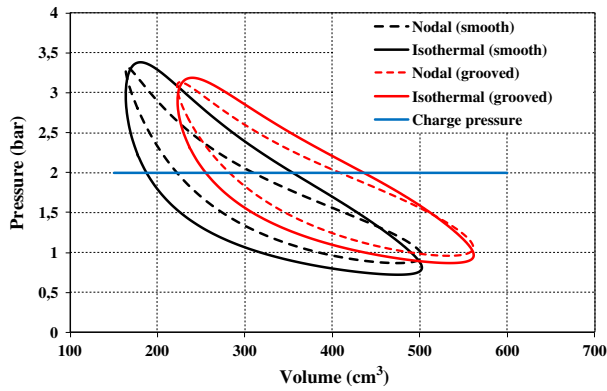


Fig. 5. Comparison of p-V diagrams obtained with grooved and smooth displacer cylinders.

While the maximum values of work obtained for the smooth displacer cylinder with convective heat transfer coefficients of 200, 300, and 400 W/m² K were 10.34, 15.52, and 20.63 J, respectively, the corresponding values obtained with the use of the grooved displacer cylinder were 14.09, 23.92, and 31.88 J. The engine work increases with increases in the working fluid mass. However, after a certain working fluid mass, the engine work decreases due to an insufficient heat transfer surface area. The engine work increases when using the grooved displacer cylinder due to its high heat transfer surface area.

The variations of engine power with engine speed for different convective heat transfer coefficients are given in Fig. 7. Maximum engine powers for grooved and smooth displacer cylinders were obtained at an engine speed of 500 rpm. The maximum engine powers obtained with the smooth displacer cylinder were 86.30, 129.48 and 172.63 W, respectively, for convective heat transfer coefficients of 200, 300, and 400 W/m² K. The maximum engine powers obtained with the grooved displacer cylinder were 131.94, 199.40 and 265.87 W, respectively, for convective heat transfer coefficients of 200, 300, and 400 W/m² K. Engine power increased up to an engine speed of 500 rpm and then decreased. The heat-exchange time decreased with increases in engine speed. Therefore, the amount of heat that is transferred to the working fluid becomes insufficient.

The variations in engine power and torque obtained with the use of the smooth displacer cylinder are given in Fig. 8. The tests were conducted at a hot-end temperature of 673 K and a cold-

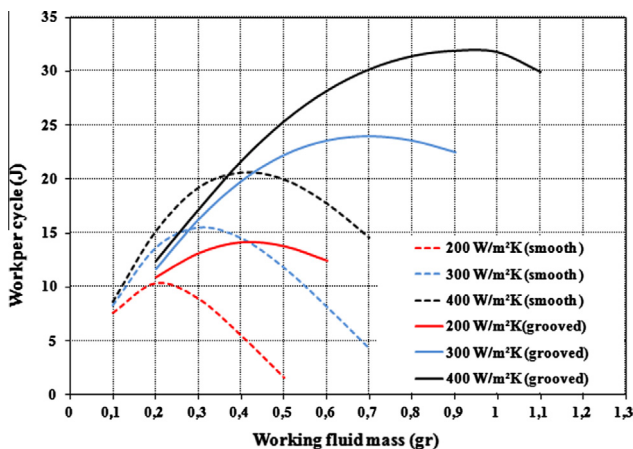


Fig. 6. The variation of work per cycle with working fluid for different convective heat transfer coefficients.

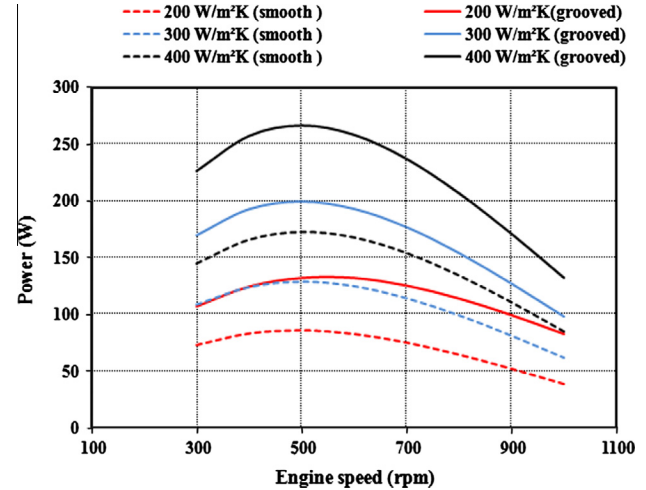


Fig. 7. The variation of engine power with engine speed for different convective heat transfer coefficients.

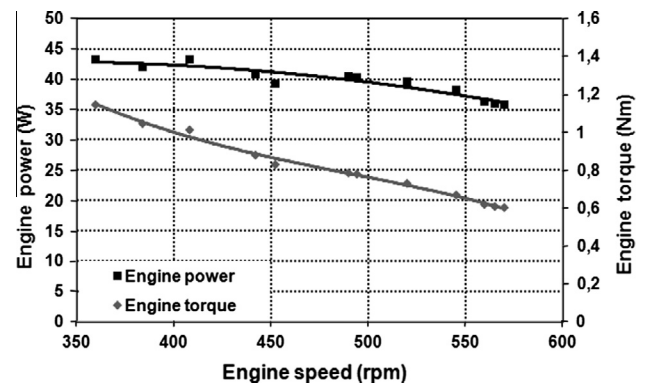


Fig. 8. The variation of engine power and torque with engine speed for the smooth displacer cylinder.

end temperature of 300 K, with atmospheric air as the working fluid. Maximum engine power and cyclic work were found to be 43.2 W and 6.35 J, respectively, at an engine speed of 300 rpm. The test conditions were carried out in the nodal analysis program

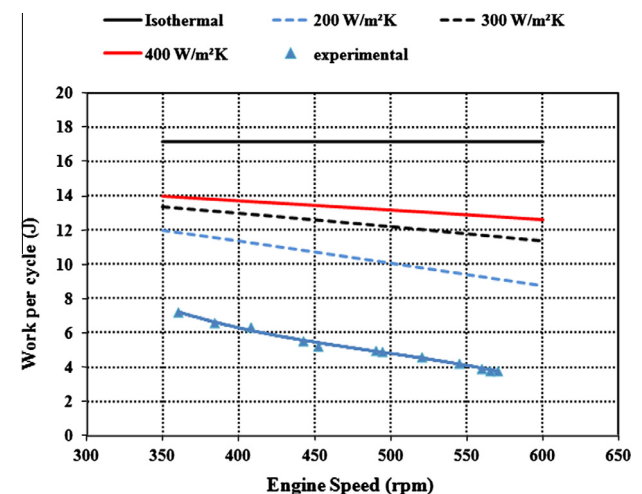


Fig. 9. Comparison of theoretical and experimental results obtained with atmospheric condition.

and the convective heat transfer coefficient was estimated as $104 \text{ W/m}^2 \text{ K}$.

The comparison of experimental results and theoretical results is shown in Fig. 9. The values of cyclic work were calculated by isothermal and nodal analysis conducted with convective heat transfer coefficients of 200, 300, and $400 \text{ W/m}^2 \text{ K}$. The difference between the experimental and theoretical results was quite small at low engine speeds. The difference between the experimental and theoretical values of cyclic work increased with engine speed. In the mathematical model, heat transfer between the working fluid and solid surfaces of the engine is taken into consideration. The heat-exchange time decreases with increasing engine speed. In addition, viscous flow losses and friction losses increase. These problems cause differences between the mathematical model and experimental results.

6. Conclusions

In this study, the effect of the grooved displacer cylinder on engine performance was investigated from kinematic and thermodynamic points of view. The maximum engine powers obtained with the use of the smooth displacer cylinder were 86.30, 129.48, and 172.63 W , for convective heat transfer coefficients of 200, 300, and $400 \text{ W/m}^2 \text{ K}$, respectively, while the corresponding engine powers obtained with the grooved displacer cylinder were 131.94, 199.40, and 265.87 W . Since both the heat transfer surface area and the regenerator volume increased with the use of the grooved displacer cylinder, engine performance was increased significantly. The maximum engine power obtained was 43.2 W at 408 rpm in experiments conducted under atmospheric conditions using the smooth displacer cylinder. By nodal analysis conducted under experimental conditions, the convective heat transfer coefficient was estimated to be $104 \text{ W/m}^2 \text{ K}$.

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